СИСТЕМНЫЙ АНАЛИЗ И УПРАВЛЕНИЕ
SYSTEM ANALYSIS AND PROCESSING OF KNOWLEDGE

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THE MODEL OF PRODUCTION PROCESS OF THE PARTY OF THE SUBJECTS OF LABOUR

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Abstract
The article discusses the construction of the model of streaming production line with the constraints on the technological trajectory of subjects of labour. The work shows the influence of the subject of labour movement trajectory, which is related to the limited maximum capacity of the operating storage. It analyses constraint that is associated with the serial order of subjects of labour processing. The equation for the trajectory of the regulatory process is built, taking into account the constraints on the trajectory of the subjects of labour, which can be used for closing the balance equations of PDE-models of streaming production lines.
Keywords: Euler's equation; the production line; mass production; work in progress; the Lagrange formalism; technological trajectory; production line; PDE model.

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МОДЕЛЬ ПРОИЗВОДСТВЕННОГО ПРОЦЕССА ОБРАБОТКИ ПАРТИИ ПРЕДМЕТОВ ТРУДА

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Аннотация
В статье рассматривается методика построения модели производственной поточной линии при наличие технологических ограничений на движение предмета труда по технологическому маршруту. Проанализировано влияние двух основных ограничений: ограничения, связанного с максимальной емкостью технологических накопителей и ограничения, связанного с заданным технологией производства последовательным порядком обработки предмета труда. Построена функция Лагранжа, описывающая движение партии предметов труда в технологическом пространстве состояний, получены уравнения движения отдельных предметов труда вдоль технологического маршрута. Записанные уравнения Эйлера могут быть использованы для замыкания балансовых уравнений PDE-моделей производственных поточных линий.
Ключевые слова: уравнение Эйлера; производственная линия; массовое производство; незавершенное производство; формализм Лагранжа; технологическая траектория; PDE модель.

INTRODUCTION

Regular changes of the products range causes the need of enterprises to design the effective systems of production control based on modern economic-mathematical models describing production phenomena [1-4]. Modern industrial enterprise functioning in conditions of uncertainty requires the usage of new, highly effective ways and methods of economic management [1-6]. The streaming models occupy an important place among the models of streaming production lines [7-12]. The models of streaming production lines using partial differential equations (PDE-models) are singled out into special class among other production models. [6]. One of the ways, which makes it possible to close the system of balance equations of PDE model, is an approach that uses the averaged equation of subjects of labour motion on the technological route. The subject of labour moves on the technological route under different technological constraints. The restriction on the capacity of inter-operational storage drive is one of the types of constraints. The subject of labour, which passed technological process, moves to the inter-operational storage drive (fig. 1). The overflow of inter-operational storage drive leads to step of the production process [13-16]. Inter-operational storage drive acts as a buffer, which smoothers out the intermittent work and a synchronicity in the production rate of technological equipment [17-21].

Another constraint on the subject of labour movement trajectory is the fact, that the subject of labour processing cannot be started until the processing of the previous subject of labour hasn't been finished [22, 23]. Waiting in line for the technological processing, the subject of labour is in the inter-operational storage drive. It causes the fact that the technological trajectories of subjects of labour do not intersect. The technological trajectory of the subject of labour acts as the restriction for the trajectory of the next subject of labour.

The goal of the study is to make the models of the streaming production lines, with provision for the constraints imposed on the movement trajectories of subjects of labour by the production system.

The relevance of the work contains the making the formalised description of the production system in the form by Lagrange and directly the derivation of the equation of subjects of labour motion on a technological trajectory with the constraints caused by the interaction of labour subjects with each other and with technological equipment. The equation of subjects of labour movement is used to close the balance equations of PDE models. The precision of its construction greatly affects the accuracy of PDE models of streaming production lines [20, 21, 24]. This fact causes to the actuality of the research made in this work.

Problem statement. On the streaming production line (fig. 1) we need to manufacture a batch of similar parts in the amount of N pieces. The production line includes m technological positions, on each of which m-th technological operation is running. Each position, starting from the second one, contains the storage drive, processing module and devices for parts movement between the positions (fig. 1). To describe the work of the streaming production line we use designations $\Delta S_{m,\,\psi}$ (RUB) – the cost of the resources, transferred by the processing module while operation; $\Delta\tau_m$ (h) effective time of the subject of labour processing on the m-th operation [1].

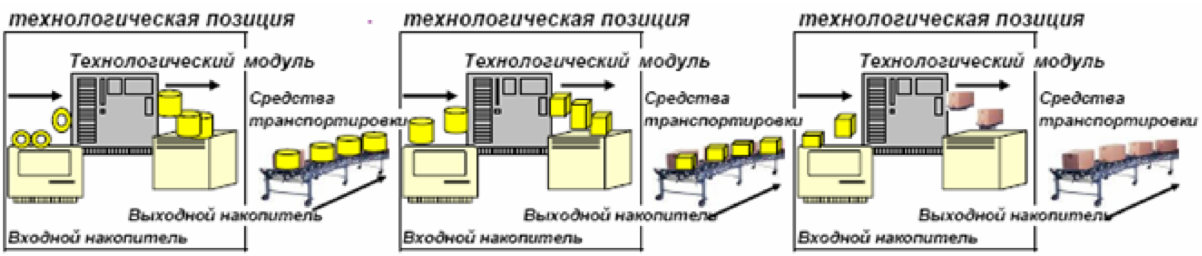

Fig. 1. The structure of the streaming production line

The total cost of the resources, transferred by the processing modules of the subject of labour after the execution of the m-th operation, is calculated as the sum of the costs of the technological operations: $S_{m,\,\psi} = \sum_{k=1}^{m} \Delta S_{k,\,\psi}$, RUB). The normative tempo of the subjects of labour processing on the m-th module is $[\chi]_{1m,\psi} = \dfrac{1}{\Delta\tau_m}$ (pcs. /h). For the free streaming line the characteristic of the separate part is tempo of co-processing $[\chi]_{1m_\Sigma,\psi}$, which represents the temp of subjects of labour exit from m-th module of the streaming production line $[\chi]_{1m_\Sigma,\psi} = \min\{[\chi]_{1k,\psi},\ k = 1, m\}$ (pcs. /h). We suppose that technological modes of the subject of labor processing are specified and constant during the period of the production cycle of batch processing. The state of the subject of Labor will be characterised by the phase technological space coordinates (S, μ) [6].

The values of the parameters of the condition and the position of the j-th subject of labour at the moment of time t we will determine with the help of the cost of technological resources transferred on it $S_j(t)$ (RUB) and the intensity of the transferring of the technological resources $\mu_j(t)$ (rub. /h). The principle of the transferring of resources is based on the peculiarities of the operation mode of the technological process [25]. The fig.2 shows the graph of the function $\Delta S_{\psi}(t)$ describing the transferring of resources on the subject of labour as the result of the operation for different types of processing. The left graph (fig. 1) is determined by the fact that during the time $N_m \cdot \Delta\tau_m$ of the N_m subjects of labour staying in the storage drive before the m-th technological module the cost of resources is not transferred to them.

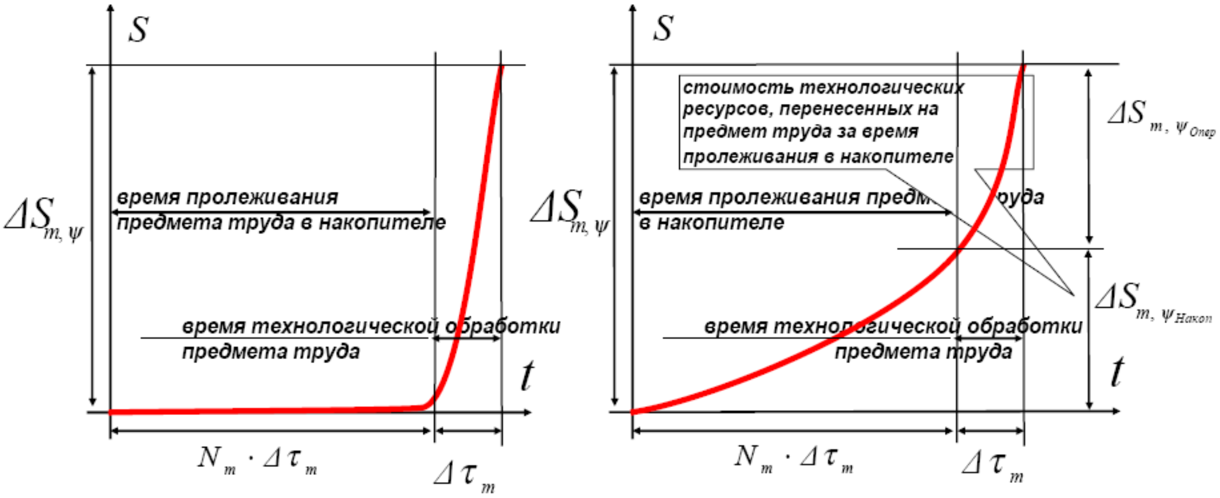

Fig. 2. Transferring of the technological recourses on the subject of labour

Table 1

Parameters of the technological operations

Operation number	1st	2nd	3rd	4th	5th	7th	8th	9th
$\Delta S_{m,\psi}$, RUB	1	5	1	8	1	14	2	9
$\Delta\tau_m$, h	2	1	3	2	9	7	4	2
$S_{m,\psi} = \sum_{k=1}^{m} \Delta S_{k,\psi}$, RUB	1	6	7	15	16	30	32	41
$\tau_m = \sum_{k=1}^{m} \Delta\tau_k$, RUB	2	3	6	8	17	24	28	30
$[\chi]_{1m,\psi} = \frac{1}{\Delta\tau_m}$, pcs/h	0,5	1	0,33	0,5	0,11	0,14	0,25	0,5
$[\chi]_{1m_\Sigma,\psi} = \min\{[\chi]_{1k,\psi},\ k=1,m\}$, pcs/h	0,5	0,5	0,33	0,33	0,11	0,11	0,11	0,11
$\mu_{m,\psi} = \frac{\Delta S_{m,\psi}}{\Delta\tau_m}$, rub/h	0,5	5	0,33	4	0,11	2	0,5	4,5
Entrance storage drive capacity, $N_{m\psi\,Max}$, pcs	10	3	5	7	8	8	8	8

These operations include the operation of machining and assembling. The second type of operations is determined by the fact that during the subjects of labor staying in the storage drives, the cost of resources is not transferred on them (left graph, Fig 1).

Let’s consider the subject of labor movement on the technological route and linking operations, during which the resources transferring happens during processing of the subjects of labor. Let's assume that, if the part is in a storage drive, its cost does not increase. After the end of the technological processing of the subject of labor on m-th technological module, its cost increases on value $\Delta S_{m,\psi}$.

Duration of transferring of the subject of labour from the storage drive to the technological position is much less than the duration of the operation. The trajectory of the subject of labour movement on technological route for free (unoccupied) streaming production line is determined by the Euler equation:

$$\frac{d\mu}{dt} = \mu_\psi(S)\frac{\partial\mu_\psi(S)}{\partial S}, \qquad \frac{dS}{dt} = \mu . \tag{1}$$

The equation (1) allows the solving in analytical form for some functions $\mu_\psi(S)$. For functions of the type $\mu_\psi(S) = \sqrt{2 \cdot S}$ the equation of the subject of labour movement trajectory is defined by the quadratic function $S(t) = \frac{1}{2}\cdot t^2$, for functions type $\mu_\psi(S) = 1$ – by a linear function $S(t) = t$. Let's consider the movement of the batch of 5 parts on the route that consists of 8 operations, each of which is characterised by values $\Delta S_{m,\psi}$ и $\Delta\tau_m$ (table 1). It is anticipated that the processing time for m-th operation is deterministic.

1. THE BATCH 0F THE SUBJECT LABOUR PROCESSING ON THE FREE STREAMING PRODUCTION LINE

The technological trajectory of the first subject of labour, moving on the free streaming production line with the normative tempo $[\chi]_{1m,\psi} = \frac{1}{\Delta\tau_m}$ (pcs/h) (fig. 3) satisfies the equation (1) presented in Fig. 4. While constructing the technological trajectory of the function $\mu_{m,\psi} = \mu_{m,\psi}(S_{m,\psi})$ (table 1) is approximated by a continuous smooth function $\mu_\psi(S)$. The characteristic points of the technological trajectory of the first subject of labor for the free streaming production line are defined by integer coordinates ($S_{m,\psi}, \tau_m$): A_1(0,0); B_1(1,2); C_1(6,3); D_1(7,6); G_1(15,8); E_1(16,17); K_1(30,24); H_1(32,28); Q_1(41,30) (fig. 4, fig. 5). The letters, with the exception of A_1(0, 0), correspond to the coordinates of the end of the m-th operation. The index at the bottom indicates the subject of labour that the trajectory belongs to. If after the end of the (m-1) th operation the subject of labour still stays in the storage drive before the m-th operation, the characteristic point of the beginning of the subject of labour processing on the m-th operation will be denoted with letters in italic. The segment, connecting points C_5 (6.11) and c_5 (6.15), forms the part of the trajectory of which the 5-th subject of labour is waiting for processing (fig. 6). The segment of the technological trajectory, connecting points G_5 (15.18) and g_5 (15.44), is similar (fig.6). While the movement of the first subject of labour on the free streaming production line is described by equation (1), the movement of the second and subsequent subjects of labour is limited by the condition of the previous processing. The first constraint is that it is possible to start the processing of the j-th subject of labour at the m-th only after the end of the processing of the (j-1) th subject of labour. At any time for two points $S_{j-1}(t_1)$ and $S_j(t_1)$ of the trajectories of the subjects of labour following after each other (fig. 5) the next relation is valid:

$$S_{j-1}(t) = S_j(t) + S_{12}(S_j). \tag{2}$$

Analogically, when $S_{j-1}(t_1) = S_j(t_2)$ for two points $S_{j-1}(t_1)$ and $S_j(t_2)$ of the trajectories of the successive subjects of labour:

$$\mathrm{t}_j = t_{j-1} + t_{12}(S_{j-1}),\ S_{j-1}(t_1) = S_j(t_2) \tag{3}$$

The first constraint can be written in the form of inequality

$$S_{j-1}(t - \tau_{\psi 12}(S_j)) \geq S_j(t). \tag{4}$$

The function $\Delta\tau_m(S_m)$ is represented by a continuous function, so that point for the point $S = S_m$

$$\tau_{\psi 12}(S_m) = \Delta\tau_m(S_m). \tag{5}$$

According to (2)-(4) trajectories of (j-1) th and j-th subject of labour cannot cross. The constraint is that processing of the j-th subject of labour on the m-th operation must be completed later than the beginning of processing of the $(j - \mathrm{N}_{m,\psi\ \mathrm{Max}})$-th subject of labour on the (m+1)th operation with entrance storage drive $\mathrm{N}_{(m+1),\psi\ \mathrm{Max}}$. When processing of the $\mathrm{N}_{(m+1),\psi\ \mathrm{Max}}$ subject of on the (m+1)th operation on the entrance storage drive with capacity $\mathrm{N}_{(m+1),\psi\ \mathrm{Max}}$ there is free space which we get as the result of fact that the j-th subject of labour is under processing on the m-th operation. The constraint connected with the ultimate storage drive capacity is written in the form of inequality

$$S_{\text{j-N}_{m,\psi\ Max}}(t) \geq S_j(t). \tag{6}$$

We suppose that the j-th subject of labour waiting for the end of the processing of the $\left(\text{j}-\text{N}_{m,\psi\ \text{Max}}\right)$-th subject of labour on the (m+1) th operation is in m-th module.

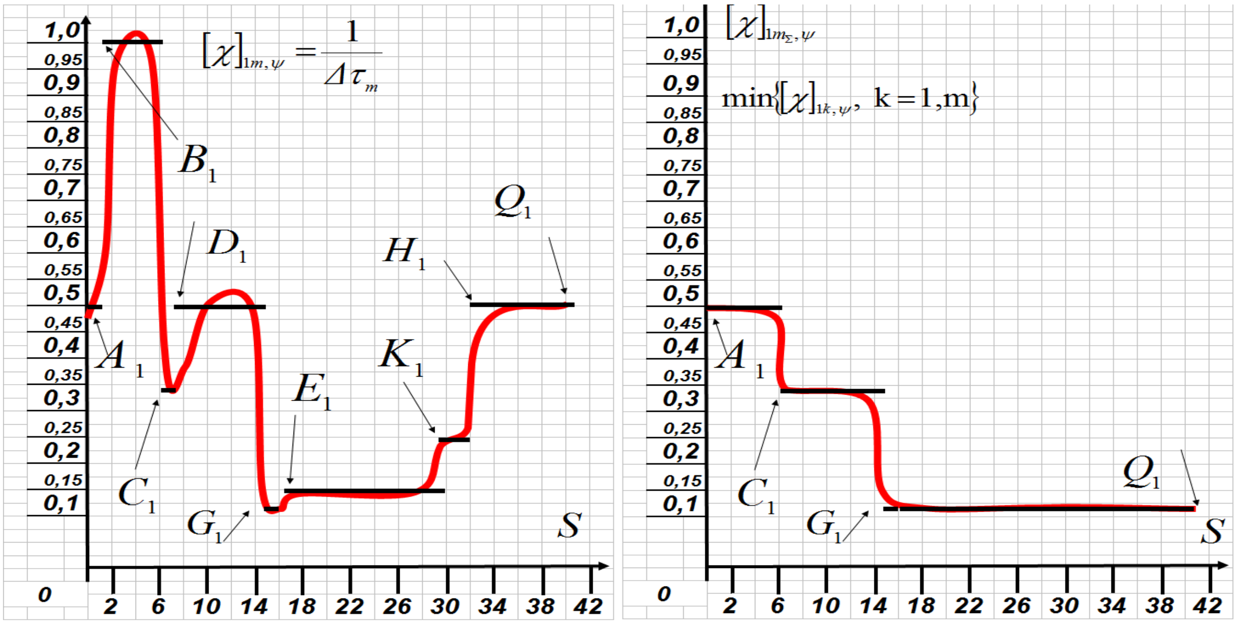

Fig. 3. Rate of the batch processing: a-normative; b-while co-processing

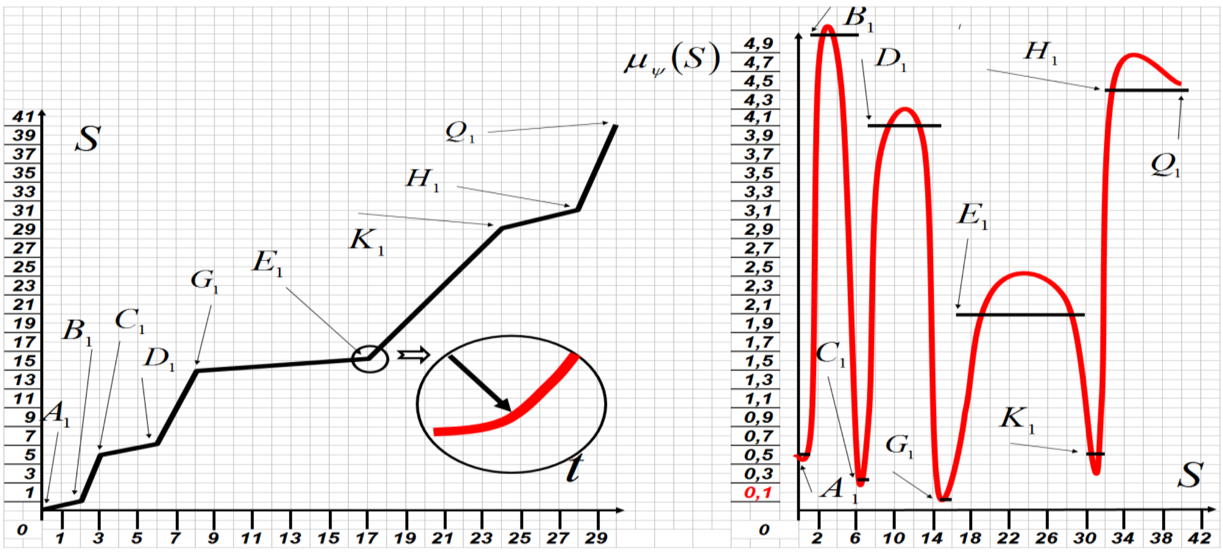

Fig. 4. Technological trajectory of the first subject of labour

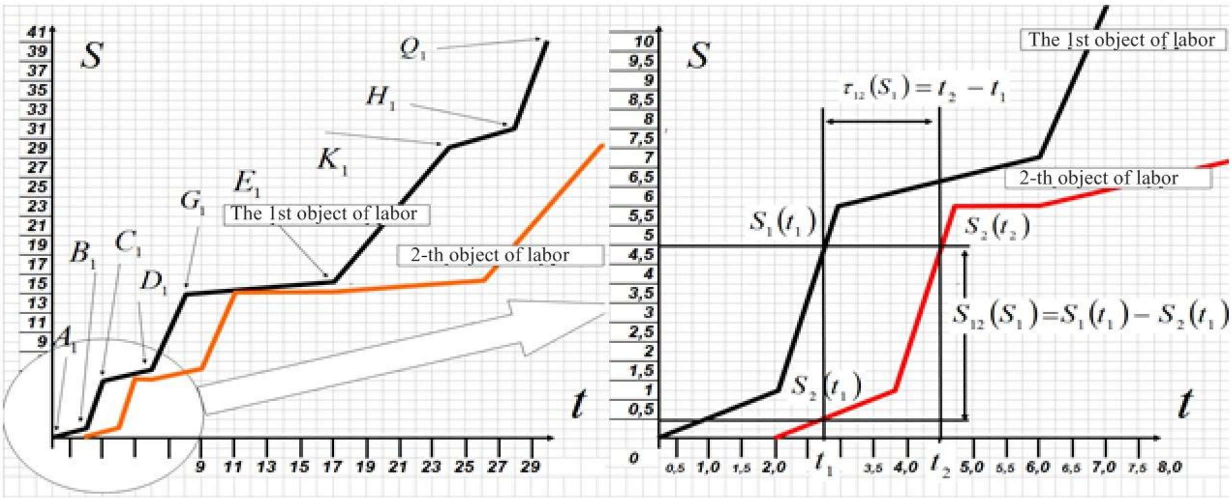

Fig. 5. Constraints on the trajectories imposed by the conditions of processing

The trajectory of the J-th subject of labour in phase technology space satisfies the equations

$$\frac{d}{dt}\frac{\partial L}{\partial \mu_{j_i}} = \frac{\partial L}{\partial S_j}, \qquad \frac{dS_j}{dt} = \mu_j, \ (j=1..N) \tag{7}$$

$$S_j(t_{0j}) = 0, \qquad \mu_j(t_{0j}) = \mu_{0j}, \tag{8}$$

with the constraints imposed on the technological trajectories connected with the sequence of the subjects of labour processing and storage drive capacity:

$$S_{j-1}(t-\tau_{\psi 12}(S_j)) \geq S_j(t), \ S_{\text{j-N}_{m,\psi}\ \text{Max}}(t) \geq S_j(t). \tag{9}$$

Lagrange function is defined as

$$L = J(S_j, \mu_j) + \lambda_1 \cdot \{S_{j-1}(t-\tau_{\psi 12}(S_j)) - S_j(t)\} + \lambda_2 \cdot \{S_{\text{j-N}_{m,\psi}\ \text{Max}}(t) - S_j(t)\}, \tag{10}$$

where $J(S_j, \mu_j) = \mu_j^2 + \mu_\psi^2(S_j)$, t_{0j} - a time of the beginning of the j-th subject of labour processing.

We assume that the transferring of the resources on the individual separate operation occurs only on one subject of labour. It is not possible to process two following after each other subjects of labour on the m-th operation at the same time. The equations of movement for –j-th subject of labour in phase space considering (5)-(10) have the following form:

$$\frac{d\mu_j}{dt} = \mu_\psi(S_j)\frac{\partial \mu_\psi(S_j)}{\partial S} + \lambda_1 \cdot \left\{\mu_{j-1}(t-\tau_{\psi 12}(S_j)) \cdot \frac{d(t-\tau_{\psi 12}(S_j))}{dS_j} - 1\right\} - \lambda_2,$$

$$\frac{dS_j}{dt} = \mu_j, \tag{11}$$

$$S_j(t_{0j}) = 0, \qquad \mu_j(t_{0j}) = \mu_{0j}, \qquad (j=1..N) \tag{12}$$

$$S_{j-1}(t-\tau_{\psi 12}(S_j)) \geq S_j(t), \ \lambda_1 \geq 0,$$

$$\lambda_1 \cdot \{S_{j-1}(t-\tau_{\psi 12}(S_j)) - S_j(t)\} = 0, \tag{13}$$

$$S_{\text{j-N}_{m,\psi}\ \text{Max}}(t) \geq S_j(t), \qquad \lambda_2 \geq 0,$$

$$\lambda_2 \cdot \{S_{\text{j-N}_{m,\psi}\ \text{Max}}(t) - S_j(t)\} = 0 \tag{14}$$

$$\lambda_1 \equiv 0 \text{ when } j-1 \leq 0 \ \text{ and } \lambda_2 \equiv 0 \text{ when}$$

$$j - \mathrm{N}_{m,\psi\ \text{Max}} \leq 0. \tag{15}$$

From the condition by Kun-Tacker the conditions of additional state slackness results. It means that Lagrange multiplier λ_k equals zero if the constraint is performed as strict inequality, and the corresponding Lagrange multiplier is positive, if the constraint is performed as equality. Solving the system of equations (11)-(15) for the batch of five subjects of labour, moving on the free streaming production line with equipment parameters (table 1) and initial conditions

$$\mathrm{S}_j(\varDelta\tau_1\cdot(j-1))=0\,,\qquad \mu_j(\varDelta\tau_1\cdot(j-1))=\mu_\psi(0),\qquad (j=1..5) \tag{16}$$

are presented in fig. 6.

The trajectory of movement of the j-th subject of labour on the segment $[\mathrm{A}_j\ldots\mathrm{g}_j]$ is determined by the constraint (4) $S_{j-1}(t-\tau_{\psi 12}(S_j))\geq S_j(t)$ that leads to the fact that subjects of labour wait before the third and the fifth operations (fig. 8).

After the fifth operation subjects of labour movement doesn't have constraints. The equation describing the motion of subjects of labour after the fifth operation is:

$$\frac{d\mu_j}{dt}=\mu_\psi(\mathrm{S}_j)\frac{\partial\mu_\psi(\mathrm{S}_j)}{\partial S}\,,\qquad \frac{\mathrm{dS}_j}{\mathrm{dt}}=\mu_j\,. \tag{17}$$

$$\mathrm{S}_j(\tau_5+\varDelta\tau_5\cdot(j-1))=\mathrm{S}_1(\tau_5)=S_{5,\psi}\,,\ (j=1..5) \tag{18}$$

Conditions (16), (18) are defined by the tempo of co-processing $[\chi]_{1m_\Sigma,\psi}$ of a batch of subjects of labour after the fifth operation $[\chi]_{1m,\psi}>[\chi]_{1m_\Sigma,\psi}$, $m>5$ (fig. 3), that determines the time intervals of subjects of labor exit from the fifth and the subsequent operations (fig. 6). If we suggest for this batch of subjects of labour the initial conditions

$$\mathrm{S}_j(\varDelta\tau_5\cdot(j-1))=0\,,\qquad \mu_j(\varDelta\tau_5\cdot(j-1))=\mu_\psi(0)\,,\qquad (j=1..5) \tag{19}$$

determined by the tempo of co-processing of the batch $[\chi]_{1m_\Sigma,\psi}$, the constraints (13), (14) transform into strict equality with Lagrange multipliers λ_k =0. With that, the system of equations (17), (18), describing the movement of subjects of labour along the route of a free streaming production line, will take the form

$$\frac{d\mu_j}{dt}=\mu_\psi(\mathrm{S}_j)\frac{\partial\mu_\psi(\mathrm{S}_j)}{\partial S}\,,\frac{\mathrm{dS}_j}{\mathrm{dt}}=\mu_j\,, \tag{20}$$

Solving the system of equations (19), (20) is presented in Fig. 7. Technological trajectories of subjects of labour are shifted along the time axis on the value $\Delta\tau_5$, corresponding to the processing time of the subject of lab our on the 5-th technological operation. With the original data (19) the solving of the system of equations (20) corresponds to the situation of the processing of the j-th subject of labour on 5-th operation without waiting in the entrance drive.

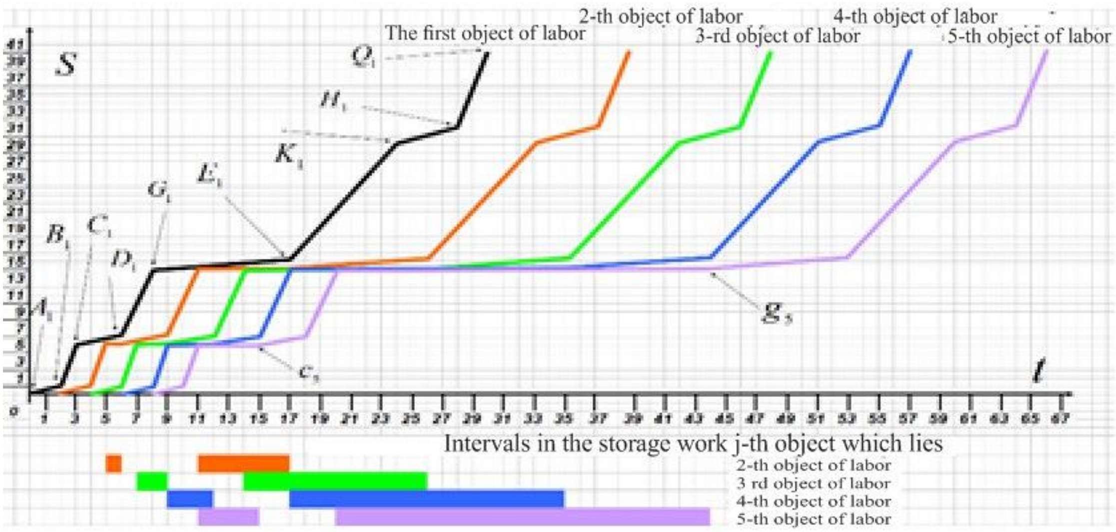

Fig. 6. The batch of subjects of labour movement trajectories with waiting in the drive

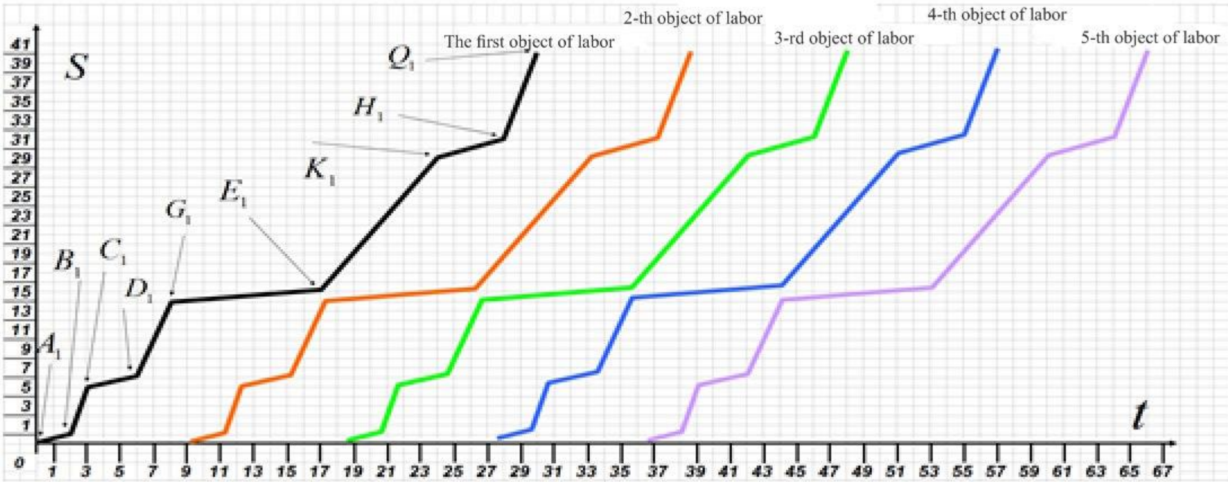

Fig. 7. The batch of subjects of labour movement trajectories without waiting in the drive

It should be noted that the production cycle time for the case with initial conditions (19) and (16), (12) remained unchanged. Thus, the following initial conditions

$$S_j\left(\frac{1}{[\chi]_{1M_\Sigma,\psi}}\cdot(j-1)\right)=0,$$

$$\mu_j\left(\frac{1}{[\chi]_{1M_\Sigma,\psi}}\cdot(j-1)\right)=\mu_\psi(0), \quad (j=1..N) \tag{21}$$

do not affect the time of production cycle of the parts.

With increasing of the time between the start of the technological processing of subjects of labour on the first technological operation to $\left([\chi]_{1M_\Sigma,\psi}\right)^{-1}$ the quantity of the subjects of labour in storages doesn't increase. When $\Delta\tau_1\to\left([\chi]_{1M_\Sigma,\psi}\right)^{-1}$ the processing of the batch of the subjects of labour can be carried out without input storage devices (fig. 7).

The input of the subjects of labour on the first technological operation is determined by condition (21)

THE PROCESSING OF THE BATCH OF SUBJECT LABOUR ON BUSY PRODUCTION LINE

We examine the movement of the batch of parts (consisting of 5 items) on the technological route of the production streaming line, busy with the processing of a previous batch. The technological route consists of 8 operations, each of which is characterised by values $\Delta S_{m,\psi}$ and $\Delta\tau_m$ (table 1). The batch consisting of 10 subjects of labour is processed on the streaming production line. The subject of lab our route consists of 6 operations, each of which is characterised by values $\Delta S_{m,\psi}$ and $\Delta\tau_m$ (table 2). Before processing of the batch of 5 subjects of labour, the streaming production line is busy with the processing of the batch of 10 subjects of labour. The trajectories of movement of the last subjects of the labour of the previous batch are constrained for the trajectories of movement of subjects of Labour of the following batch, entering the processing. The equations of movement of the j-th subject of labour received from the batch entering the processing have following forms:

$$\frac{d\mu_j}{dt}=\mu_\psi(S_j)\frac{\partial\mu_\psi(S_j)}{\partial S}+\lambda_1\cdot\left\{\mu_{j-1}\left(t-\tau_{\psi 12}(S_j)\right)\cdot\frac{d\left(t-\tau_{\psi 12}(S_j)\right)}{dS_j}-1\right\}-\lambda_2,\quad \frac{dS_j}{dt}=\mu_j,$$

$$S_j(t_{нj})=0,\quad \mu_j(t_{нj})=\mu_{нj},\quad (j=1..N),$$

$$S_{j-1}\left(t-\tau_{\psi 12}(S_j)\right)\geq S_j(t),\quad \lambda_1\geq 0,\quad \lambda_1\cdot\left\{S_{j-1}\left(t-\tau_{\psi 12}(S_j)\right)-S_j(t)\right\}=0,$$

$$S_{j-N_{m,\psi\ Max}}(t)\geq S_j(t),\qquad \lambda_2\geq 0,\quad \lambda_2\cdot\left\{S_{j-N_{m,\psi\ Max}}(t)-S_j(t)\right\}=0.$$

In the considered case, while determining the trajectories of subjects of labour from the batch entering the processing, the index for $S_{j-1}\left(t-\tau_{\psi 12}(S_j)\right)$ and $S_{j-N_{m,\psi\ Max}}(t)$ can be less than zero or equal to zero. It means that the trajectory of movement of the subject of labour from the batch entering the processing is limited by the trajectory of movement of the subjects of labour from the previous batch (fig. 8).

Table 2

Parameters of the technological operations

Operation number	1st	2nd	3rd	4th	5th	6th
$\Delta S_{m,\psi}$, RUB	3	6	2	8	14	3
$\Delta\tau_m$, h	5	3	6	6	1	8
$S_{m,\psi}=\sum_{k=1}^{m}\Delta S_{k,\psi}$, RUB	3	9	11	19	33	36
$\tau_m=\sum_{k=1}^{m}\Delta\tau_k$, h	3	8	14	20	21	29
$[\chi]_{1m,\psi}=\frac{1}{\Delta\tau_m}$, pcs. /h	0,2	0,33	0,17	0,17	1	0,12
$[\chi]_{1m_\Sigma,\psi}=\min\{[\chi]_{1k,\psi},\ k=1,m\}$, pcs./h	0,2	0,2	0,17	0,17	0,17	0,12
$\mu_{m,\psi}=\frac{\Delta S_{m,\psi}}{\Delta\tau_m}$, hrn. /h	0,6	2	0,33	1,33	14	0,38
Entrance storage drive capacity, $N_{m\psi\ Max}$, pcs	5	2	3	4	4	4

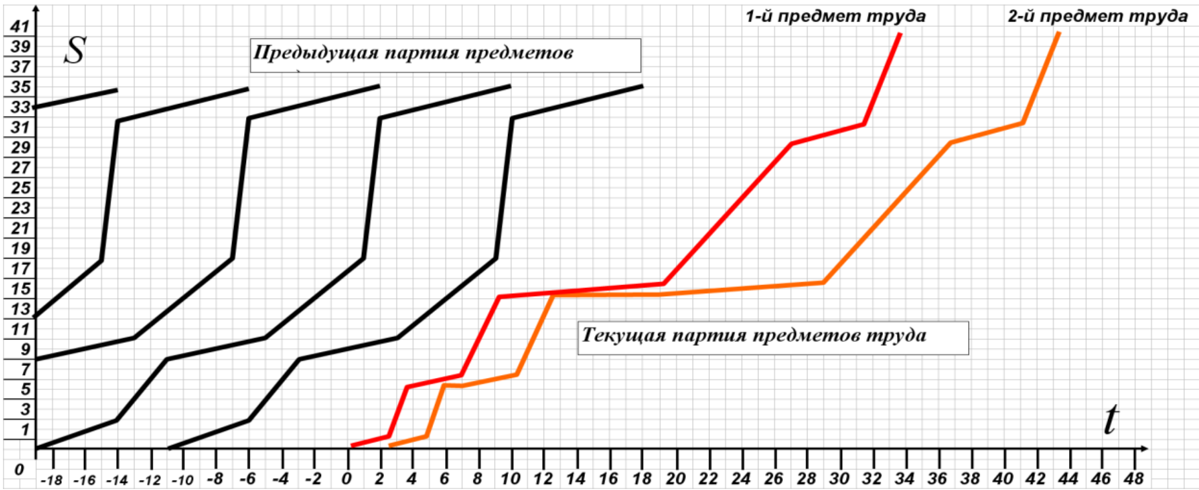

Fig. 8. Initial trajectories of the subjects of labour for busy

CONCLUSION

Final part. In the work we built a domain-technical description of the production process, based on conservation laws characterising the process of transfer of technological resources on the subject of labour, and the law of spatiotemporal structure of the technological process. It is shown that parameters of the managed production process are the variables which behaviour in general, is caused by the processes of transferring of technological resources on the subject of labour. The state of parameters of the production process is determined by the status of a great number of parameters of subjects of labour that are on the different processing stages along the route of technological operations. For the derivation of the unsteady-state equations of state of parameters of streaming line functioning in transient modes, we got the equation subjects of labour motion.

The following things were made in this work. A model of processing batches on the free streaming line was considered. We analysed the constraints that are imposed on technological trajectories of subjects of labour in production technology. The system of equations by Lagrange is obtained to describe changes of the properties of the batch of the subject of labour in the phase space, taking constraints into account. We defined the scope of values for the initial parameters conditions of subjects of labour, admitting analytical solution of a system of equations by Lagrange. It is shown that if we follow the conditions of continuous processing of subjects of labour on one or few technological operations for a period of time, that is much longer than the effective time of the processing, the initial conditions do not affect the tempo of production.

As developing of the tasks about the batch of products processing on the free streaming line, the problem of processing of subjects of labour on the production line, busy with the processing of the previous batch, was considered. We got the system of equations by Lagrange to describe the movement of the subjects of labour batch in the phase space, taking into account the imposed constraints. Similarly to the task of processing of batch of products on the free streaming line, it is shown that for the period of time, much longer than the effective time of the processing, the initial conditions do not affect the tempo of the production under the circumstance of continuous operating of the modules for each technological operation.

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